# The Stress Intensity Factor

In most cases, the Inglis stress concentration equation (eqn.(9.5)) can be approximated as

$$\sigma_{max} = \sigma_0\left[1 + 2\left(\frac{c}{r}\right)^{1/2}\right] \approx 2\sigma_0\left(\frac{c}{r}\right)^{1/2} \qquad (10.2)$$

Since a critical stress is presumably needed at the crack tip to open up the atomic planes, it follows that crack propagation is expected when

$$\sigma_0\sqrt{c} \;\geq\; \text{critical value} \qquad (10.3)$$

where the critical value is expected to be constant for a given material, but to vary between materials (since $r$, and probably the critical stress level as well, will differ for different materials). In the 1950's, Irwin proposed the concept of a ***stress intensity factor***, $K$, such that

$$K = \sigma_0\sqrt{\pi c} \qquad (10.4)$$

The stress intensity factor, which has units of MPa √m, scales with the level of stress at the crack tip (although it does not allow the absolute value to be established). Fracture is expected when $K$ reaches a critical value, $K_c$, the ***critical stress intensity factor***, which is often termed the ***fracture toughness***.

### *Uniting the Stress and Energy Approaches*

It is clear that there are parallels between $K$ reaching a critical value, $K_c$, and $G$ reaching a critical value of $G_c$. The magnitude of $K$ can be considered to represent the crack driving force, analogous to $G$. Consider again the Griffiths energy criterion

$$G \geq G_c, \quad \text{ie} \quad \pi\left(\frac{\sigma_0^2 c}{E}\right) \geq G_c \quad \therefore \sigma_0\sqrt{\pi c} \geq \sqrt{EG_c} \qquad (10.5)$$

It can be seen that this is actually in a similar form to eqn.(10.4). It follows that

$$K = \sqrt{EG} \quad \text{and} \quad K_c = \sqrt{EG_c} \qquad (10.6)$$